

BREW BY THE

Have you ever wondered just how much wallop your favorite homemade

NUMBERS —

beverage packs, alcoholwise and caloriewise? Have you ever heard your

ADD UP WHAT'S

brewing buddies talk about apparent extract and real attenuation and wondered

IN YOUR BEER

what all the hubbub was about? It's not as hard to understand as you might think.

IN THE FOLLOWING ARTICLE I WILL DEVELOP EQUATIONS TO DESCRIBE MANY DIFFERENT FACETS OF HOMEBREWING. In each section, I first give an equation that is as accurate as possible, including the effects of all of the important parameters. This equation is sometimes rather complicated, but the idea is to program it into a computer spreadsheet and never look at it again. Then, where possible, I simplify the equations using appropriate assumptions so that calculations can be made on the fly in the brewery or kitchen.

One thing I should explain at the outset is curve fitting. There are many occasions when a functional relationship for a set of data is needed but not available. Curve fitting (also called regression or least-squares fitting) is nothing more than a mathematical way to draw the best curve through a set of experimental data points. In this article I use quadratic ($a + bx + cx^2$) and cubic ($a + bx + cx^2 + dx^3$) functions to draw the curves.

SPECIFIC GRAVITY

Specific gravity is defined as the density of a substance relative to the density of water. This is not quite sufficient as a definition because the density of water varies with temperature. Above 39 degrees F, water expands as it heats up, making it necessary to specify the temperature when indicating a specific gravity.

The most common way for a homebrewer to measure specific gravity is to use a hydrometer. A hydrometer measures the density of a fluid in units of the density of water at some reference temperature (the temperature at which the hydrometer is calibrated). In other words, the measurement you get from your hydrometer is really

$$\text{Measured SG} = \frac{\text{density of wort at temperature } T}{\text{density of water at reference temperature}} \quad (1)$$



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For most of the hydrometers available to homebrewers the reference temperature is 60 degrees F, which is the value that will be used in the rest of this section.

The measured specific gravity value must be temperature corrected before it is meaningful. The ideal way to do this would be to convert the density of the wort to the reference temperature, like this:

$$\text{Ideal SG} = \frac{\text{density of wort at } 60^{\circ}\text{F}}{\text{density of water at } 60^{\circ}\text{F}} \quad (2)$$

Modifying the wort density to account for temperature is not that easy, however, because we don't have tabulated data for the variation of wort density with temperature for every possible wort composition. Fortunately, the temperature coefficients of expansion for wort and water are practically the same, making the density ratio roughly invariant to temperature. Making this assumption leads us to this equation:

$$\text{Corrected SG} = \frac{\text{density of wort at } T}{\text{density of water at } T} \quad (3)$$

$$= \left(\frac{\text{density of wort at } T}{\text{density of water at } 60^{\circ}\text{F}} \right) \left(\frac{\text{density of water at } 60^{\circ}\text{F}}{\text{density of water at } T} \right)$$

$$= (\text{Measured SG at } T) (\text{SG correction factor})$$

This correction factor is solely a function of the density of water at various temperatures, which is well-known. I made a curve fit to some data from a couple of sources (De Clerck, Weast) that yielded the SG correction factor as a function of measuring temperature (T in Fahrenheit):

$$\text{SG correction factor} = 1.00130346 - 1.34722124 \times 10^{-4} T + 2.04052596 \times 10^{-6} T^2 - 2.32820948 \times 10^{-9} T^3 \quad (4)$$

This equation is accurate over the entire region from 32 degrees F to 212 degrees F.

If you're putting this equation in a spreadsheet, the most accurate way to do it would be to multiply your measured specific gravity by the correction factor evaluated at the temperature at which the SG measurement was made, as shown in Equation 4. However, for the times when it is easier to just add or subtract a point or two, we can make the following approximation. Both the measured SG and the correction factor are numerically very close to one. If we represent them by one plus a small number (e_{SG} or e_{CF}), then

$$\begin{aligned} \text{Corrected SG} &= (\text{Measured SG at T}) (\text{SG correction factor}) \\ &= (1 + e_{SG}) (1 + e_{CF}) \\ &= 1 + e_{SG} + e_{CF} + e_{SG} e_{CF} \end{aligned} \tag{5}$$

Because both e terms represent small quantities, multiplying them together makes a number that is very small with respect to the other terms and can be neglected. This yields

$$\begin{aligned} \text{Corrected SG} &\cong 1 + e_{SG} + e_{CF} \\ &= (\text{Measured SG at T}) + (\text{SG correction factor} - 1) \end{aligned} \tag{6}$$

So, instead of multiplying by the correction factor, you can just add the correction factor minus one. Table 1 gives the additive correction factors (in points, see SG in glossary for explanation) as a function of the measuring temperature. Using an additive correction factor is not as accurate as the multiplicative correction factor, but it is easier and is adequate for most circumstances.

EXTRACT

Since the early days of brewing, brewers (and drinkers) have been concerned about the strength of their concoctions. One measure of wort strength is how much material has been extracted from the malted grains into solution by the mashing and lautering processes. The term "extract" refers to the weight percent of dissolved materials in the wort. Weight percent is percent by weight. For example, if you have five pounds of sugar in a solution that weighs 100 pounds total, the solution is 5 weight percent sugar. (Incidentally, this is also 5 °Plato.)

The only difficulty with this is that there is not a good way to measure the amount of dissolved materials, short of evaporating your wort until you have dry malt extract, measuring the weight of the powder and then dividing by the weight of the original solution. One thing that you can measure is the specific gravity, which is the density of the solution relative to the density of water. But there is still a problem: the dissolved materials are made up of fermentable sugars, non-fermentable sugars, proteins and other goodies, with proportions that vary from wort to wort. How can the relationship between weight percent of solids and specific gravity be determined if the identity of the solids isn't even known?

In 1843, Carl Joseph Napoleon Balling determined a way around this problem. He noticed that the specific gravity of a wort increased with the weight percent of dissolved materials in almost the same

TABLE 1: SPECIFIC GRAVITY TEMPERATURE CORRECTION

Measuring Temperature (F)	Addition to Specific Gravity in points	Measuring Temperature (F)	Addition to Specific Gravity in points
32	-0.83	105	6.96
40	-0.97	110	8.08
45	-0.84	115	9.26
50	-0.69	120	10.50
55	-0.38	125	11.80
60	0.00	130	13.16
65	0.53	140	16.07
70	1.05	150	19.15
75	1.69	160	22.45
80	2.39	170	25.93
85	3.17	180	29.59
90	4.01	190	33.40
95	5.01	200	37.35
100	5.91	212	42.42

NOTE: Subjecting a room-temperature hydrometer to 212-degree-F temperatures can result in a broken hydrometer.

manner as if the dissolved materials were entirely sucrose. He could do all of his experiments with pure sucrose and water, and they would be a good approximation to beer worts. After making up sugar solutions, Balling developed a table relating the density of the solution (at 17.5 degrees C) to the weight percent of sugar. He measured the weight percent in degrees Balling (°B), defined as the number of grams of sugar per 100 grams of wort. Several years later, around 1900, Dr. Fritz Plato corrected some slight mistakes and developed his own set of tables, calling the corrected unit a degree Plato (°P). To convert back and forth between extract (E) in degrees Plato and specific gravity (SG), you can use these equations:

$$E = -668.962 + 1262.45 \text{ SG} - 776.43 \text{ SG}^2 + 182.94 \text{ SG}^3 \tag{7}$$

$$\text{SG} = 1.00001 + 0.0038661 E + 1.3488 \times 10^{-5} E^2 + 4.3074 \times 10^{-8} E^3 \tag{8}$$

These equations are accurate cubic fits that I did to Plato's data (Timmermans) over the range of 0 to 33 °P, which covers specific gravities between 1.000 and 1.144. A simpler formula that is good for most applications is:

$$E = 1000 (\text{SG} - 1) / 4 \tag{9}$$

In other words, just take the number of specific gravity points and divide by four to get the extract value in degrees Plato. For example, if you measured the specific gravity of your wort to be 1.065, then the simple formula (Equation 9) gives an extract of 16.25 °P, while the cubic formula (Equation 7) gives an answer of 15.88 °P.

But there's still a little more to extract than this. If you measure the specific gravity before fermentation starts and convert

that number to degrees Plato, that's called the original extract (OE). If you take a measurement after fermentation is finished, that's called the apparent extract (AE). The reason it isn't the real extract is that your beer is no longer just a solution of solids and water. Now you have alcohol in there too, and alcohol is less dense than water (specific gravity of 0.794 at 15 degrees C) so it changes all the neat equations that we've just come up with. Just as before, the hard way to determine the real extract (RE) of your beer would be to boil it until all of the alcohol is boiled off, replace the volume with water and then measure the specific gravity. Fortunately, Balling once again comes to the rescue with an empirical relationship between the real extract, the apparent extract and the original extract (De Clerck):

$$\begin{aligned} q &= 0.22 + 0.001 \text{ OE} \\ \text{RE} &= (q \text{ OE} + \text{AE}) / (1 + q) \end{aligned} \tag{10}$$

The variable q is called the attenuation coefficient, but you can just think of it as an intermediate value. Many sources quote this equation in the form $\text{RE} = 0.8192 \text{ AE} + 0.1808 \text{ OE}$, but this form assumes that q is calculated at an OE of zero, which is not very accurate. For our simplified version, we'll calculate q assuming a reasonable OE of 12.5 °P, which gives us

$$\text{RE} = 0.8114 \text{ AE} + 0.1886 \text{ OE} \tag{11}$$

Let's say our beer has finished fermenting and we have measured the specific gravity to be 1.014. The cubic formula for extract (Equation 7) gives an apparent extract of 3.57 °P, but the simple formula does very well and gives a value of 3.50 °P. Calculating the real extract using the most accurate RE formula (Equation 10) and the cubic extract formula gives 5.92 °P. Making the same calculation with the simple version of both formulas (Equations 9 and 11) gives 5.90 °P. The difference in real extract calculated by these two methods is small, but the two methods show greater differences in subsequent calculations. If you're not interested in extreme accuracy, then the simple formulas are probably adequate. This is especially true of beers with original gravities less than 1.070, because the two formulas only diverge significantly for high specific gravities. You might want to go through all the complicated calculations for a barleywine or a mead (or if you're putting all of this in a spreadsheet), but for regular beers the simple divide-by-four rule is all you need.

The simple formulas will be used in the rest of the examples, with the results from the more accurate formulas in parentheses for comparison.

ATTENUATION

A closely related term that is often used to describe a yeast strain or the dryness of a beer is attenuation. Attenuation is simply the percentage of sugar that has been converted to alcohol. Attenuation comes in two forms, just like the final extract value. Apparent attenuation (AA) is calculated using the apparent extract value:

$$\text{AA} = \frac{(\text{OE} - \text{AE})}{\text{OE}} \times 100\% \tag{12}$$

Real attenuation (RA) is calculated using the real extract value:

$$\text{RA} = \frac{(\text{OE} - \text{RE})}{\text{OE}} \times 100\% \tag{13}$$

For our sample beer, the apparent attenuation is 78.5 percent (77.5 percent) and the real attenuation is 63.7 percent (62.7 percent). Most beers will have a real attenuation that falls in the 60 to 80 percent range.

ALCOHOL CONTENT

One might think that the alcohol percentage of the final beer would be directly proportional to the difference in the original and final (real) extract values. After all, the chemical equation for the conversion of a monosaccharide to ethanol is



so the weight of the sugar molecule (180 amu) should be converted into the weight of two ethanol molecules (92 amu) and two carbon dioxide molecules (88 amu). This would give us an equation for the alcohol percent by weight (A%w) of:

$$\text{A}\%w = (\text{OE} - \text{RE}) \frac{92}{180} = \frac{(\text{OE} - \text{RE})}{1.9565} \tag{15}$$

The fly in the ointment here is that fermentation is not that simple. It's a biological process with all kinds of intermediate products and side reactions that don't lead to our desired result. Balling comes through for us again with an empirical formula for the alcohol content (De Clerck):

$$\text{A}\%w = \frac{\text{OE} - \text{RE}}{2.0665 - 0.010665 \text{ OE}} \tag{16}$$

If we insert the simple extract equation (Equation 9) and the simplified version of Balling's equation for real extract (Equation 11), we can derive a relationship for the alcohol content as a function of the original and final specific gravities:

$$\text{A}\%w = \frac{76.08 (\text{OG} - \text{FG})}{1.775 - \text{OG}} \tag{17}$$

Either of these equations can be converted to alcohol percent by volume (A%v) by the following formula:

$$\text{A}\%v = \text{A}\%w (\text{FG} / 0.794) \tag{18}$$

where 0.794 is the specific gravity of ethanol. For our sample brew, the alcohol percentages are 5.46 percent (5.25 percent) by weight

TABLE 2: ALCOHOL PERCENT BY WEIGHT (USING MOST ACCURATE EQUATIONS)

Original Specific Gravity	Final Specific Gravity												
	0.990	0.995	1.000	1.005	1.010	1.015	1.020	1.025	1.030	1.035	1.040	1.045	1.050
1.030	4.17	3.63	3.10	2.57	2.05	1.53	1.01	0.51	0.00	—	—	—	—
1.035	4.69	4.15	3.62	3.09	2.56	2.04	1.52	1.01	0.50	0.00	—	—	—
1.040	5.22	4.68	4.14	3.61	3.08	2.55	2.03	1.52	1.01	0.50	0.00	—	—
1.045	5.75	5.21	4.67	4.13	3.60	3.07	2.55	2.03	1.51	1.01	0.50	0.00	—
1.050	6.28	5.73	5.19	4.65	4.12	3.59	3.06	2.54	2.02	1.51	1.00	0.50	0.00
1.055	6.82	6.26	5.72	5.17	4.64	4.10	3.58	3.05	2.53	2.02	1.51	1.00	0.50
1.060	7.35	6.80	6.25	5.70	5.16	4.62	4.09	3.56	3.04	2.52	2.01	1.50	1.00
1.065	7.89	7.33	6.78	6.23	5.68	5.14	4.61	4.08	3.55	3.03	2.52	2.01	1.50
1.070	8.42	7.86	7.31	6.75	6.21	5.67	5.13	4.60	4.07	3.54	3.02	2.51	2.00
1.075	8.96	8.40	7.84	7.28	6.73	6.19	5.65	5.11	4.58	4.06	3.53	3.02	2.50
1.080	9.50	8.94	8.37	7.82	7.26	6.72	6.17	5.63	5.10	4.57	4.04	3.52	3.01
1.085	10.05	9.48	8.91	8.35	7.79	7.24	6.70	6.15	5.62	5.08	4.56	4.03	3.51
1.090	10.59	10.02	9.45	8.88	8.33	7.77	7.22	6.68	6.14	5.60	5.07	4.54	4.02
1.095	11.14	10.56	9.99	9.42	8.86	8.30	7.75	7.20	6.66	6.12	5.59	5.06	4.53
1.100	11.69	11.11	10.53	9.96	9.39	8.83	8.28	7.73	7.18	6.64	6.10	5.57	5.04
1.105	12.24	11.65	11.07	10.50	9.93	9.37	8.81	8.26	7.71	7.16	6.62	6.09	5.56
1.110	12.79	12.20	11.62	11.04	10.47	9.90	9.34	8.78	8.23	7.69	7.14	6.60	6.07

TABLE 3: CALORIES PER 12-OUNCE BOTTLE OF BEER (USING MOST ACCURATE EQUATIONS)

Original Specific Gravity	Final Specific Gravity												
	0.990	0.995	1.000	1.005	1.010	1.015	1.020	1.025	1.030	1.035	1.040	1.045	1.050
1.030	93.6	96.0	98.3	100.7	103.1	105.5	107.9	110.2	112.6	—	—	—	—
1.035	110.1	112.5	114.8	117.2	119.6	122.0	124.3	126.7	129.1	131.5	—	—	—
1.040	126.7	129.0	131.4	133.8	136.1	138.5	140.9	143.2	145.6	148.0	150.4	—	—
1.045	143.3	145.6	148.0	150.3	152.7	155.0	157.4	159.8	162.1	164.5	166.9	169.3	—
1.050	159.9	162.2	164.6	166.9	169.3	171.6	174.0	176.4	178.7	181.1	183.4	185.8	188.2
1.055	176.6	178.9	181.2	183.6	185.9	188.3	190.6	193.0	195.3	197.7	200.0	202.4	204.7
1.060	193.3	195.6	197.9	200.3	202.6	204.9	207.3	209.6	212.0	214.3	216.6	219.0	221.3
1.065	210.0	212.3	214.7	217.0	219.3	221.6	224.0	226.3	228.6	231.0	233.3	235.6	238.0
1.070	226.8	229.1	231.4	233.7	236.1	238.4	240.7	243.0	245.3	247.7	250.0	252.3	254.7
1.075	243.6	245.9	248.2	250.5	252.8	255.2	257.5	259.8	262.1	264.4	266.7	269.1	271.4
1.080	260.5	262.8	265.1	267.4	269.7	272.0	274.3	276.6	278.9	281.2	283.5	285.8	288.1
1.085	277.3	279.6	281.9	284.2	286.5	288.8	291.1	293.4	295.7	298.0	300.3	302.6	304.9
1.090	294.3	296.6	298.8	301.1	303.4	305.7	308.0	310.3	312.6	314.9	317.2	319.5	321.7
1.095	311.2	313.5	315.8	318.1	320.4	322.6	324.9	327.2	329.5	331.8	334.0	336.3	338.6
1.100	328.3	330.5	332.8	335.1	337.3	339.6	341.9	344.2	346.4	348.7	351.0	353.3	355.5
1.105	345.3	347.6	349.8	352.1	354.4	356.6	358.9	361.1	363.4	365.7	367.9	370.2	372.5
1.110	362.4	364.7	366.9	369.2	371.4	373.7	375.9	378.2	380.4	382.7	385.0	387.2	389.5

and 6.98 percent (6.71 percent) by volume. Table 2 shows the alcohol percent by weight as a function of original and final specific gravities calculated with the most accurate equations (Equations 7, 10 and 16) above.

CALORIE CONTENT

Before all that alcohol goes to your head, let's calculate how your homebrew adds to your beer belly. The following equations all give calories (C) per one 12-ounce bottle of beer. First, there's a contribution due to the residual sugar (extract) in the beer:

$$C_{\text{ext}} = 3.55 \text{ FG (3.8) RE} \quad (19)$$

The 3.8 factor is the number of calories per gram of sugar, the final specific gravity converts from grams of solution to grams of water, and the factor of 3.55 is the number of grams of water in a 12-ounce bottle, divided by 100 to cancel with the implicit percent of the real extract. There's also a contribution because of the alcohol present:

$$C_{\text{alc}} = 3.55 \text{ FG (7.1) A\%w} \quad (20)$$

The 7.1 factor here is indicative of alcohol's higher number of calories per gram. Finally, there's a small contribution because of the protein in the beer:

$$C_{\text{pro}} = 3.55 \text{ FG (4.0) (0.07) RE} \quad (21)$$

The 4.0 factor represents the calories per gram of protein, and the percentage of protein has been estimated at 7 percent of the percentage of sugar. This estimate is a median value for protein estimates I have seen in the literature, which range from 5 to 10 percent of the real extract value. The calorie per gram values are from De Clerck. The total number of calories in your homebrew is then:

$$\begin{aligned} C &= C_{\text{ext}} + C_{\text{alc}} + C_{\text{pro}} \\ &= 3.55 \text{ FG [3.8 RE + 7.1 A\%w + (4.0) (0.07) RE]} \\ &= 3.55 \text{ FG (4.08 RE + 7.1 A\%w)} \end{aligned} \quad (22)$$

If we convert the calorie equation to a function of specific gravities using both of Balling's approximations and the simple equation for extract (Equations 9, 11 and 17) we get:

$$C = 3621 \text{ FG} \left[(0.8114 \text{ FG} + 0.1886 \text{ OG} - 1) + 0.53 \frac{\text{OG} - \text{FG}}{1.775 - \text{OG}} \right] \quad (23)$$

For our example beer this gives us a calorie count of 226.5 (221.2) per 12-ounce bottle. Table 3 shows the calorie content as a function of original and final specific gravities calculated with the most accurate equations (Equations 7, 10, 16 and 22) above.

Okay, now, hang on tight for some fast and furious approximations. First, note that a gram of sugar gives 3.8 calories, but when it converts to alcohol it gives roughly $7.1 (92 / 180) = 3.63$ calories.

This means that the sugar doesn't lose a lot of calories by converting to alcohol, and therefore the calorie count is primarily a function of the original specific gravity. We can take advantage of this fact by calculating the calories of the unfermented wort (setting the $\text{FG} = \text{OG}$), and realizing that our estimate will be a little high. Better yet, we can make an educated guess about the final specific gravity, setting it equal to one-fourth of the point value of the original specific gravity:

$$\text{FG} = \frac{\text{OG} - 1}{4} + 1 \quad (24)$$

Second, that $(1.775 - \text{OG})$ term in the denominator of Equation 23 is going to give us trouble, so let's set that particular OG to a mid-range value of 1.050. Making those approximations and fiddling around with the numbers a bit yields:

$$C = 851 (\text{OG} - 1) (\text{OG} + 3) \quad (25)$$

For our example beer this gives a calorie count of 224.8 (221.2) per 12-ounce bottle. For most beers, this equation will be a reasonable approximation of the number of calories.

CARBONATION LEVEL

If you are a homebrew bottler instead of a homebrew kegger, you're probably a little bit jealous of all the control that a kegger has over carbonation levels. If you're a kegger, it's relatively easy: you just put your beer (whether or not it has finished fermenting) into a keg and adjust the temperature to the desired serving temperature and adjust your pressure so that the carbonation level is what you want. You determine the desired carbonation level by reading the "volumes of CO_2 " off a chart as a function of the temperature and pressure. You can always readjust things if they are not to your liking. (Okay, it's not really that simple, but you get my point.)

A bottler has a more difficult life when it comes to carbonation. Once the cap is on, everything is fixed. If you overcarbonate you have to chill all your bottles down and drink them as fast as you can. If you undercarbonate, your beer suffers from a lack of aroma, lack of tingly mouthfeel and overall aesthetics suffer. Clearly, this situation could benefit from a little more control.

First, what exactly is a "volume of CO_2 "? The number of volumes of CO_2 is a measure of the amount of dissolved carbon dioxide. It is equal to the volume occupied by the carbon dioxide if it were taken out of solution and put at standard temperature and pressure (STP = 32°F or 0°C and 1 atmosphere) divided by the volume of the beer. In other words, if you took all the carbon dioxide out of your five-gallon batch of beer, changed it to STP and got 10 gallons of gas, the CO_2 level in the original beer was 2 volumes. The amount of carbon dioxide that will dissolve is a fixed quantity which depends on temperature and pressure (and also the other things in solution, but that effect is negligible for our purposes).

TABLE 4: CARBON DIOXIDE EQUILIBRIUM CONCENTRATION

Temperature (F)	Volumes of CO ₂ In Solution	Temperature (F)	Volumes of CO ₂ In Solution
32	1.71	60	0.99
35	1.61	65	0.91
40	1.46	70	0.83
45	1.32	75	0.78
50	1.20	80	0.73
55	1.09		

A desirable carbonation level for most beers is 2.2 to 2.6 volumes of CO₂. Miller recommends 1.8 to 2.2 volumes for British ales, 2.5 volumes for lagers and German ales, 2.6 to 2.8 volumes for American beers and 3.0 volumes for wheat beers and fruit ales.

Now that our target carbonation level is set, we need to determine our starting point. If your beer has been bubbling away in a carboy since the last time you racked it, it has an overpressure of carbon dioxide at one atmosphere. This means that a considerable amount of carbon dioxide is already in solution in your beer. The amount that is in solution (CD_{init} in volumes) is a function of temperature (T, in Fahrenheit). I fit a function to some empirical data (Linke) to give this relationship:

$$CD_{init} = 3.0378 - 5.0062 \times 10^{-2} T + 2.6555 \times 10^{-4} T^2 \quad (25)$$

Table 4 shows that this function varies from 1.71 to 0.73 volumes over the range of possible bottling temperatures, which indicates that determining the amount of dissolved CO₂ at the start of bottling is crucial.

The next step is calculating how much sugar is necessary to get from the initial carbon dioxide level to the target level. I want to point out here that measuring your priming sugar by volume (the old three-fourths cup per five-gallon batch rule) is not very accurate. We're going to need a certain mass of sugar to get the desired mass of CO₂, so if you measure by volume you'll need to know the density to determine the mass. I've seen density estimates for corn sugar that varied between 133 and 193 grams per cup. I measured it myself to be 151 grams per cup. In the calculations that follow, it is assumed that the sugar is measured by weight instead of volume, and I strongly recommend that you measure it that way too.

Given a weight of priming sugar (PS) in grams and the volume of beer (VB) in gallons, we can estimate the amount of carbon dioxide generated through fermentation (CD_{gen}):

$$CD_{gen} = PS \left(\frac{88g \text{ CO}_2}{180g \text{ C}_6\text{H}_{12}\text{O}_6} \right) \left(\frac{1}{7.4287g \text{ CO}_2 / \text{gallon at STP}} \right) \left(\frac{1}{VB} \right) \\ = 6.5811 \times 10^{-2} \frac{PS}{VB} \text{ volumes} \quad (27)$$

This formula makes use of the simplistic chemical equation of fermentation (Equation 14) and assumes that 100 percent of the sugar is fermented. This assumption is valid for corn sugar and table sugar, but if you're priming with something else, like honey or dry malt extract, you should include a factor for the fraction of the priming substance that is fermentable, and increase your priming rate accordingly.

The total amount of carbon dioxide in our primed and conditioned beer is then:

$$CD = CD_{gen} + CD_{init} \quad (28) \\ = 6.5811 \times 10^{-2} \left(\frac{PS}{VB} \right) + 3.0378 - 5.0062 \times 10^{-2} T + 2.6555 \times 10^{-4} T^2$$

Inverting this formula we can get an equation for the weight of priming sugar:

$$PS = 15.195 VB (CD - 3.0378 + 5.0062 \times 10^{-2} T - 2.6555 \times 10^{-4} T^2) \quad (29)$$

So there's the complicated formula. Table 5 gives values from this formula for various combinations of carbonation level and temperature. What would an "average" case look like? Let's assume that you have five gallons of fully fermented beer at 65 degrees F and you want to have 2.5 volumes of CO₂ in the final product. Running the numbers gives a priming sugar weight of 121 grams. If you insist on having a volume of priming sugar to work with, this comes out to 0.8 cups (using the density for corn sugar that I measured). Lest you think that it always comes out close to three-quarters of a cup, here's another example: This time we'll assume that you've been making a lager and your beer is waiting to be primed at 40 degrees F, with everything else the same. The result for this case is 79 grams of priming sugar, considerably less than before.

I should mention here the caveat that if your beer hasn't finished fermenting, none of this analysis is accurate and you may have significant amounts of sugar left that will create large amounts of carbon dioxide. The amount of priming sugar that is normally used will only raise the specific gravity by 0.002, so you can see that it is imperative that there is not an extra bit of fermentable sugar hanging around. In practice, termination of fermentation is easy to discern by watching your fermentation lock or taking specific gravity readings.

CONCLUSION

I hope that these explanations have made the confusing world of real extracts and apparent attenuations a little bit clearer. Using the equations in this article (and keeping good records) should enable you to duplicate your beer successes and to avoid reinventing your beer duds. And now that you've been armed with information about how your homebrew affects your head and body, you will be able to make a better decision about whether or not to have that next beer.

TABLE 5: PRIMING SUGAR WEIGHT (IN GRAMS) FOR A FIVE-GALLON BATCH OF BEER

Temp. of beer	Desired Carbonation Level (Volumes of CO ₂)										
	1.8	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	3.0
32	7.0	22.2	29.8	37.4	45.0	52.6	60.2	67.8	75.4	83.0	98.2
35	14.4	29.6	37.2	44.8	52.4	60.0	67.6	75.1	82.7	90.3	105.5
40	25.8	41.0	48.6	56.2	63.8	71.4	79.0	86.6	94.2	101.8	117.0
45	36.3	51.5	59.1	66.7	74.3	81.9	89.4	97.0	104.6	112.2	127.4
50	45.7	60.9	68.5	76.1	83.7	91.3	98.9	106.5	114.1	121.7	136.9
55	54.1	69.3	76.9	84.5	92.1	99.7	107.3	114.9	122.5	130.1	145.3
60	61.5	76.7	84.3	91.9	99.5	107.1	114.7	122.3	129.9	137.5	152.7
65	68.0	83.2	90.8	98.3	105.9	113.5	121.1	128.7	136.3	143.9	159.1
70	73.4	88.6	96.2	103.8	111.3	118.9	126.5	134.1	141.7	149.3	164.5
75	77.8	92.9	100.5	108.1	115.7	123.3	130.9	138.5	146.1	153.7	168.9
80	81.1	96.3	103.9	111.5	119.1	126.7	134.3	141.9	149.5	157.1	172.3

NOMENCLATURE / GLOSSARY

- A%w** – Alcohol percent by weight.
- A%v** – Alcohol percent by volume.
- AA** – Apparent attenuation (%), apparent percentage of sugar that converted to alcohol.
- AE** – Apparent extract (degrees Plato), the apparent weight percent of dissolved solids in the beer, before correcting for the lower density of the alcohol.
- amu** – Atomic mass unit.
- C** – Calories in a single 12-ounce beer.
- C_{alc}** – Calories in a single 12-ounce beer attributed to alcohol.
- C_{ext}** – Calories in a single 12-ounce beer attributed to extract (residual sugar).
- C_{pro}** – Calories in a single 12-ounce beer attributed to protein.
- CD** – Total carbon dioxide concentration in the conditioned beer (volumes).
- CD_{init}** – Initial carbon dioxide concentration in the beer before priming (volumes).
- CD_{gen}** – Incremental carbon dioxide concentration caused by the fermentation of the priming sugar (in volumes).
- C₂H₅OH** – Chemical formula for ethanol, the primary alcohol in beer.
- C₆H₁₂O₆** – Chemical formula for a monosaccharide sugar (glucose).
- CO₂** – Chemical formula for carbon dioxide.
- E** – Extract (degrees Plato), the weight percent of dissolved materials in the wort.
- FG** – Final specific gravity.
- OE** – Original extract (degrees Plato).
- OG** – Original specific gravity.
- PS** – Weight of the priming sugar (grams).

- RA** – Real attenuation (%), real percentage of sugar that is converted to alcohol.
- RE** – Real extract (degrees Plato), the real weight percent of dissolved solids in the beer, after correcting for the lower density of the alcohol.
- SG** – Specific gravity (density relative to water). Specific gravity in points is equal to 1000 (SG - 1).
- STP** – Standard temperature (0 degrees C or 32 degrees F) and pressure (1 atmosphere).
- T** – Temperature of the beer (Fahrenheit).
- VB** – Volume of the beer (gallons).

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